

# Bi-large Neutrino Mixing and CP violation in an $SO(10)$ SUSY GUT for Fermion Masses

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## Abstract

We construct a simple  $SO(10)$  SUSY GUT with  $D_3$  family symmetry and low energy R parity. The model describes fermion mass matrices with 14 parameters and gives excellent fits to 20 observable masses and mixing angles in both quark and lepton sectors, giving 6 predictions. Bi-large neutrino mixing is obtained with hierarchical quark and lepton Yukawa matrices; thus avoiding the possibility of large lepton flavor violation. The model naturally predicts small 1-3 neutrino mixing,  $\sin \theta_{13} \simeq 0.05$ , and a CP violating phase  $\delta$  close to  $\pi/2$ . Among other interesting predictions is a tiny effective Majorana mass for neutrinoless double-beta decay. Leptogenesis is also possible with the decay of the lightest right-handed neutrino giving an acceptable CP violating asymmetry  $\epsilon_1$  of order  $10^{-6}$ , *and with the correct sign for the resultant baryon asymmetry*. Note, similar models with the non-abelian symmetry groups  $SU(2)$  or  $D_4$ , instead of  $D_3$ , can be constructed.

There are very few experimental indications for new physics beyond the extremely successful standard model. By far the two most exciting harbingers of new physics are the successful prediction of gauge coupling unification in supersymmetric GUTs [1] and the discovery of neutrino masses and mixing. Both of these experimental results hint at a new large mass scale of order  $10^{16}$  GeV. In addition, SUSY GUTs can alleviate the gauge hierarchy problem implying the exciting possibility for the discovery of supersymmetric partners at the LHC. It is thus important to see to what extent SUSY GUTs and neutrino masses and mixing angles are compatible.

The first problem one encounters in any theory with quark-lepton unification is the fact that flavor mixing in the quark sector is small, while for neutrinos there are two large mixing angles. In the quark sector it is well-known that hierarchical Yukawa matrices “naturally” fit the family hierarchy, as well as the hierarchy of flavor mixing evident in the CKM matrix. Moreover this can be described group theoretically in terms of the hierarchical breaking of family symmetries via the Froggatt-Nielsen mechanism [2]. In supersymmetric theories, family symmetries play a dual role. In addition to describing the hierarchy of fermion masses and mixing, they “naturally” align squark mass matrices with their quark superpartners;<sup>1</sup> thus ameliorating problems with flavor violation. In this letter, we show that bi-large neutrino mixing is naturally incorporated into SUSY GUTs with hierarchical quark and lepton Yukawa matrices; thus avoiding the possibility of large lepton flavor violation <sup>2</sup>. Given Yukawa matrices of the form

$$Y \simeq \begin{pmatrix} 0 & b & b \\ b & a & a \\ b & a & 1 \end{pmatrix} \lambda \quad (1)$$

with  $b \ll a \ll 1$  and a hierarchical right-handed neutrino mass matrix of the form

$$M_R = \begin{pmatrix} M_{R_1} & 0 & 0 \\ 0 & M_{R_2} & 0 \\ 0 & 0 & M_{R_3} \end{pmatrix} \quad (2)$$

with  $M_{R_1} \ll M_{R_2} \ll M_{R_3}$  one “naturally” obtains large neutrino mixing [7, 8, 9, 10, 11, 12, 14]. Neglecting, for example, the contribution of  $M_{R_{2,3}}$  we find, upon integrating out the lightest right-handed neutrino

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{(\lambda b v_u)^2}{M_{R_1}}. \quad (3)$$

giving maximal mixing angle for atmospheric neutrinos. It has been shown by one of us that this mechanism also gives acceptable bi-large neutrino mixing if the contributions of the lightest two neutrinos are comparable [14].

Our model is an SO(10) realization of this general mechanism. This letter is organized as follows: first we present the superpotential for the fermion mass sector of an SO(10)

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<sup>1</sup>For non-abelian family symmetries, squark and slepton mass matrices are proportional to the unit matrix prior to family symmetry breaking. Then when the family symmetry is broken, generating the hierarchy of fermion masses, the squark and slepton mass matrices also receive corrections which are aligned with their fermionic partners.

<sup>2</sup>In several recent SUSY GUT constructions, bi-large neutrino mixing is obtained with so-called *lopsided* Yukawa matrices for quarks and leptons [3]. In this approach, use is made of the SU(5) relation  $Y_e = Y_d^T$  where large, *unobservable* right-handed mixing for down quarks is converted to large left-handed mixing angles for leptons (for a review, see [4, 5]). In this case, one takes  $(Y_d)_{23} \approx (Y_d)_{33} \sim 1$ . One apparent difficulty of this approach is the large induced lepton flavor violation for processes such as  $\mu \rightarrow e\gamma$  due to large off-diagonal  $(Y_e)_{32} \approx (Y_e)_{33} \sim 1$  [6].

SUSY GUT. We then fit the low energy data using a global  $\chi^2$  analysis. The  $\chi^2$  function includes 20 low energy observables for fermion masses and mixing angles, in addition to the three low energy gauge couplings. We obtain an excellent fit to the data. Then, we present predictions for additional observables (not included in the  $\chi^2$  analysis):  $\sin^2 \theta_{13}$ , the CP violating angle  $\delta$  (and Jarlskog parameter  $J$ ), the effective neutrinoless double beta decay mass parameter  $\langle m \rangle_{\beta\beta}$ , the tritium beta decay mass parameter  $m_{\nu_e}^{eff}$ , and finally the lepton number asymmetry parameter relevant for leptogenesis,  $\epsilon_1$ . Alternate versions of the model with family symmetries  $SU(2)$  or  $D_4$  (instead of  $D_3$ ) can easily be constructed [13].

### *The model*

Consider an  $SO(10)$  SUSY GUT with a  $D_3 \times [U(1) \times Z_2 \times Z_3]$  family symmetry.<sup>3</sup> The three families of quarks and leptons are contained in three 16 dimensional representations of  $SO(10)$   $\{16_a, 16_3\}$  with  $16_a$ ,  $a = 1, 2$  a  $D_3$  flavor doublet (see appendices of Ref. [15] for details on  $D_3$ ). Consider the charged fermion sector first. Although the charged fermion sector is not the main focus of this letter it is, however, necessary to first construct a superpotential which is consistent with charged fermion masses and mixing angles, since in  $SO(10)$  the neutrino and charged fermion sectors are inextricably intertwined.

The superpotential resulting in charged fermion masses and mixing angles is given by

$$W_{ch.fermions} = 16_3 \ 10 \ 16_3 + 16_a \ 10 \ \chi_a + \bar{\chi}_a (M_\chi \chi_a + 45 \frac{\phi_a}{\hat{M}} 16_3 + 45 \frac{\tilde{\phi}_a}{\hat{M}} 16_a + \mathbf{A} \ 16_a) \quad (4)$$

where  $M_\chi = M_0(1 + \alpha X + \beta Y)$  includes  $SO(10)$  breaking vevs in the  $X$  and  $Y$  directions,  $\phi^a$ ,  $\tilde{\phi}^a$  ( $D_3$  doublets),  $\mathbf{A}$  ( $\mathbf{1}_B$  singlet) are  $SO(10)$  singlet flavon fields, and  $\hat{M}$ ,  $M_0$  are  $SO(10)$  singlet masses. The fields  $45$ ,  $A$ ,  $\phi$ ,  $\tilde{\phi}$  are assumed to obtain vevs  $\langle 45 \rangle \sim (B - L) M_G$ ,  $A \ll M_0$  and

$$\langle \phi \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} 0 \\ \tilde{\phi}_2 \end{pmatrix} \quad (5)$$

with  $\phi_1 > \phi_2$ .

The superpotential, (Eqn. 4) results in the following charged fermion Yukawa matri-

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<sup>3</sup>For related charged fermion analyses in  $SO(10)$  SUSY GUTS with  $D_3 \times U(1)$  (or  $SU(2) \times U(1)^n$ ) family symmetries, see [15] (or [16, 17, 18]). Note, the novel feature in this letter is the structure and results for neutrino masses and mixing.

ces:<sup>4</sup>

$$\begin{aligned}
Y_u &= \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon \xi \\ -\epsilon' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda \\
Y_d &= \begin{pmatrix} 0 & \epsilon' & -\epsilon \xi \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda \\
Y_e &= \begin{pmatrix} 0 & -\epsilon' & 3 \epsilon \xi \\ \epsilon' & 3 \tilde{\epsilon} & 3 \epsilon \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda
\end{aligned} \tag{6}$$

with

$$\begin{aligned}
\xi &= \phi_2/\phi_1; & \tilde{\epsilon} &\propto \tilde{\phi}_2/\hat{M}; \\
\epsilon &\propto \phi_1/\hat{M}; & \epsilon' &\sim (\mathbf{A}/M_0); \\
\sigma &= \frac{1+\alpha}{1-3\alpha}; & \rho &\sim \beta \ll \alpha.
\end{aligned} \tag{7}$$

Let us now discuss neutrino masses. In the three 16s we have three electroweak doublet neutrinos ( $\nu_a$ ,  $\nu_3$ ) and three electroweak singlet anti-neutrinos ( $\bar{\nu}_a$ ,  $\bar{\nu}_3$ ).<sup>5</sup> The superpotential  $W_{ch.fermions}$  also results in a neutrino Yukawa matrix:

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \omega & \frac{3}{2} \epsilon \xi \omega \\ \epsilon' \omega & 3 \tilde{\epsilon} \omega & \frac{3}{2} \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda \tag{8}$$

with  $\omega = 2\sigma/(2\sigma - 1)$  and a Dirac neutrino mass matrix given by

$$m_\nu \equiv Y_\nu \frac{v}{\sqrt{2}} \sin \beta. \tag{9}$$

In addition, the anti-neutrinos get GUT scale masses by mixing with three  $SO(10)$  singlets  $\{N_a, a = 1, 2; N_3\}$  transforming as a  $D_3$  doublet and singlet respectively. The full superpotential is given by  $W = W_{ch.fermions} + W_{neutrino}$  with

$$\begin{aligned}
W_{neutrino} &= \overline{16}(\lambda_2 N_a 16_a + \lambda_3 N_3 16_3) \\
&+ \frac{1}{2}(S_a N_a N_a + S_3 N_3 N_3).
\end{aligned} \tag{10}$$

We assume  $\overline{16}$  obtains a vev,  $v_{16}$ , in the right-handed neutrino direction, and  $\langle S_a \rangle = M_a$  for  $a = 1, 2$  (with  $M_2 > M_1$ ) and  $\langle S_3 \rangle = M_3$ .<sup>6</sup>

<sup>4</sup>In our notation, Yukawa matrices couple electroweak doublets on the left to singlets on the right. It has been shown in Ref. [17] that excellent fits to charged fermion masses and mixing angles are obtained with this Yukawa structure.

<sup>5</sup>In an equivalent notation, we have three left-handed neutrinos ( $\nu_{La} \equiv \nu_a$ ,  $\nu_{L3} \equiv \nu_3$ ) and three right-handed neutrinos defined by ( $\nu_{Ra} \equiv \bar{\nu}_a^*$ ,  $\nu_{R3} \equiv \bar{\nu}_3^*$ ).

<sup>6</sup>These are the most general set of vevs for  $\phi_a$  and  $S_a$ . The zero vev for  $\tilde{\phi}_1$  can be enforced with a simple superpotential term such as  $S \tilde{\phi}_a \tilde{\phi}_a$ .

We thus obtain the effective neutrino mass terms given by

$$W = \nu \, m_\nu \, \bar{\nu} + \bar{\nu} \, V \, N + \frac{1}{2} \, N \, M_N \, N \quad (11)$$

with

$$V = v_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad M_N = \text{diag}(M_1, M_2, M_3). \quad (12)$$

A simple family symmetry giving the desired form of the superpotential<sup>7</sup> is  $D_3 \times U(1) \times Z_2 \times Z_3$  where the  $D_3$  charges were defined earlier, while the  $U(1)$  charge assignments are (1 for  $16_3$ , 2 for  $16_a$ , -2 for  $N_a$ , -1 for  $N_3$ , -1 for 45, 0 for  $\overline{16}$  and  $\overline{\chi}_a$ ) and everyone else fixed by these. In addition we assign  $Z_2$  charges ( $16_3, 16_a, N_3, N_a, \overline{\chi}_a, \chi_a$ ) odd, all others even and  $Z_3$  charges  $\alpha = e^{\frac{2\pi i}{3}}$  for all fields, except 45 with charge 1. *Note, that  $Z_2$  can also be interpreted as a family reflection symmetry which guarantees an unbroken low energy  $R$  parity [19].*

The electroweak singlet neutrinos  $\{\bar{\nu}, N\}$  have large masses  $V, M_N \sim M_G$ . After integrating out these heavy neutrinos, we obtain the light neutrino mass matrix given by

$$\mathcal{M} = m_\nu \, M_R^{-1} \, m_\nu^T, \quad (13)$$

where the effective right-handed neutrino Majorana mass matrix is given by:

$$M_R = V \, M_N^{-1} \, V^T \equiv \text{diag}(M_{R_1}, M_{R_2}, M_{R_3}), \quad (14)$$

with

$$M_{R_1} = (\lambda_2 \, v_{16})^2 / M_2, \quad M_{R_2} = (\lambda_2 \, v_{16})^2 / M_1, \quad M_{R_3} = (\lambda_3 \, v_{16})^2 / M_3. \quad (15)$$

Defining  $U_e$  as the  $3 \times 3$  unitary matrix for left-handed leptons needed to diagonalize  $Y_e$  (Eqn. 6), i.e.  $Y_e^D = U_e^T \, Y_e \, U_e^*$  and also  $U_\nu$  such that  $U_\nu^T \, \mathcal{M} \, U_\nu = \mathcal{M}_D = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ , then the neutrino mixing matrix is given by  $U_{PMNS} = U_e^\dagger \, U_\nu$  in terms of the flavor eigenstate ( $\nu_\alpha$ ,  $\alpha = e, \mu, \tau$ ) and mass eigenstate ( $\nu_i$ ,  $i = 1, 2, 3$ ) basis fields with

$$\nu_\alpha = \sum_i (U_{PMNS})_{\alpha i} \, \nu_i. \quad (16)$$

For  $U_{PMNS}$  we use the notation of Ref [20] with

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2}\nu_1 \\ e^{i\alpha_2/2}\nu_2 \\ \nu_3 \end{pmatrix} \quad (17)$$

<sup>7</sup>Note, highly suppressed terms of the form  $S_a \, N_a \, N_3 \, \tilde{\phi}_a \, \phi_a^2$  are still allowed. This too can be forbidden by an additional discrete  $Z_4$  symmetry.

The  $3 \times 3$  Majorana mass matrix is of the general form discussed by many authors [14]. We have

$$\mathcal{M} = \mathcal{P}_1 \mathcal{M}_1 \mathcal{P}_1 + \mathcal{P}_2 \mathcal{M}_2 \mathcal{P}_2 + \mathcal{P}_3 \mathcal{M}_3 \mathcal{P}_3 \quad (18)$$

with

$$\mathcal{M}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\epsilon' |\omega|)^2 & -3 \epsilon' \epsilon |\xi \sigma \omega| \\ 0 & -3 \epsilon' \epsilon |\xi \sigma \omega| & (3 \epsilon |\xi \sigma|)^2 \end{pmatrix} \left( \frac{\lambda v \sin \beta}{\sqrt{2} \lambda_2 v_{16}} \right)^2 M_2; \quad (19)$$

$$\mathcal{M}_2 = \begin{pmatrix} (\epsilon' |\omega|)^2 & -3 \epsilon' |\tilde{\epsilon} \omega^2| & 3 \epsilon' \epsilon |\sigma \omega| \\ -3 \epsilon' |\tilde{\epsilon} \omega^2| & (3 |\tilde{\epsilon} \omega|)^2 & -9 \epsilon |\tilde{\epsilon} \sigma \omega| \\ 3 \epsilon' \epsilon |\sigma \omega| & -9 \epsilon |\tilde{\epsilon} \sigma \omega| & (3 \epsilon |\sigma|)^2 \end{pmatrix} \left( \frac{\lambda v \sin \beta}{\sqrt{2} \lambda_2 v_{16}} \right)^2 M_1; \quad (20)$$

$$\mathcal{M}_3 = \begin{pmatrix} (\frac{3}{2} \epsilon |\xi \omega|)^2 & |\xi| (\frac{3}{2} \epsilon |\omega|)^2 & \frac{3}{2} \epsilon |\xi \omega| \\ |\xi| (\frac{3}{2} \epsilon |\omega|)^2 & (\frac{3}{2} \epsilon |\omega|)^2 & \frac{3}{2} \epsilon |\omega| \\ \frac{3}{2} \epsilon |\xi \omega| & \frac{3}{2} \epsilon |\omega| & 1 \end{pmatrix} \left( \frac{\lambda v \sin \beta}{\sqrt{2} \lambda_3 v_{16}} \right)^2 M_3 \quad (21)$$

and

$$\mathcal{P}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Phi_\omega} & 0 \\ 0 & 0 & e^{-i(\Phi_\sigma + \Phi_\xi)} \end{pmatrix}; \quad (22)$$

$$\mathcal{P}_2 = \begin{pmatrix} e^{-i\Phi_\omega} & 0 & 0 \\ 0 & e^{-i(\Phi_\xi + \Phi_\omega)} & 0 \\ 0 & 0 & e^{-i\Phi_\sigma} \end{pmatrix}; \quad (23)$$

$$\mathcal{P}_3 = \begin{pmatrix} e^{-i(\Phi_\omega + \Phi_\xi)} & 0 & 0 \\ 0 & e^{-i\Phi_\omega} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

$\mathcal{M}_1$ ,  $\mathcal{M}_2$ ,  $\mathcal{M}_3$  are in general complex rank 1 mass matrices. However only the difference in their overall phases may be observable. Thus, there are, in principle, two new CP violating phases in the neutrino sector, in addition to the four phases already fixed by charged fermion masses and mixing angles. *We shall impose the constraint that neutrino Majorana masses  $M_i$  are all real.* This eliminates two arbitrary phases. We note that the best fits with free phases for  $M_i$  are very close numerically to our zero phase model.

*Fitting the low energy data :  $\chi^2$  analysis*

Yukawa matrices in this model are described by seven real parameters  $\{\lambda, \epsilon, \tilde{\epsilon}, \sigma, \rho, \epsilon', \xi\}$  and, in general, four phases  $\{\Phi_\sigma, \Phi_\xi, \Phi_\rho, \Phi_\epsilon\}$ . Therefore, in the charged fermion sector we have 11 parameters to explain 9 masses and three mixing angles and one CP violating phase in the CKM matrix, leaving us with 2 predictions.<sup>8</sup> Note, these parameters also determine the neutrino Yukawa matrix. Finally, our minimal

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<sup>8</sup>Of course, in any supersymmetric theory there is one additional parameter in the fermion mass matrices, i.e.  $\tan \beta$ . Including this parameter, there is one less prediction for fermion masses, but then (once SUSY is discovered) we have one more prediction. This is why we have not included it explicitly in the preceding discussion.

ansatz for the right-handed neutrino mass matrix is given in terms of three additional real parameters<sup>9</sup>, i.e. the three right-handed neutrino masses. At this point the three light neutrino masses and the neutrino mixing matrix,  $U_{PMNS}$ , (3+4 observables) are completely specified. Altogether, the model describes 20 observables in the quark and lepton sectors with 14 parameters, effectively having 6 predictions.<sup>10</sup>

In addition to the parameters describing the fermion mass matrices, we have to input three parameters specifying the three gauge couplings: the GUT scale  $M_G$  defined as the scale at which  $\alpha_1$  and  $\alpha_2$  unify, the gauge coupling at the GUT scale  $\alpha_G$ , and the correction  $\epsilon_3$  to  $\alpha_3(M_G)$  necessary to fit the low energy value of the strong coupling constant. Finally we have to input the complete set of soft SUSY breaking parameters and the value of  $\mu(M_Z)$ .<sup>11</sup> All the parameters (except for  $\mu(M_Z)$ ) are run from the GUT scale to the weak scale ( $M_Z$ ) using two (one) loop RGEs for dimensionless (dimensionful) parameters. At the weak scale, the SUSY partners are integrated out leaving the two Higgs doublet model as an effective theory. We require proper electroweak symmetry breaking. Moreover, the full set of one loop, electroweak and SUSY, threshold corrections to fermion mass matrices are calculated at  $M_Z$ . Below  $M_Z$  we use 3 loop QCD and 1 loop QED RG equations to calculate light fermion masses. More details about the analysis can be found in [23] or [24].<sup>12</sup> In addition, we self-consistently include the contributions of the right-handed neutrinos to the RG running between the GUT scale and the mass of the heaviest right-handed neutrino [25].

The  $\chi^2$  function is constructed from observables given in Table 1. Note that we over constrain the quark sector. This is due to the fact that quark masses are not known with high accuracy and different combinations of quark masses usually have independent experimental and theoretical uncertainties. Thus we include three observables for the charm and bottom quark masses: the  $\overline{MS}$  running masses ( $m_c(m_c)$ ,  $m_b(m_b)$ ) and the difference in pole masses  $M_b - M_c$  obtained from heavy quark effective theory. The same is true for observables in the CKM matrix. For example, we include  $V_{td}$  and the two CP violating observables  $\epsilon_K$  and the value for  $\sin(2\beta)$  measured via the process  $B \rightarrow J/\psi K_s$ . We thus have 16 observables in the quark and charged lepton sectors. We use the central experimental values and one sigma error bars from the particle data group [20]. In case the experimental error is less than 0.1% we use  $\sigma = 0.1\%$  due to the numerical precision of our calculation.

At present only four observables in the neutrino sector are measured. These are the two neutrino mass squared differences,  $\Delta m_{31}^2$  and  $\Delta m_{21}^2$ , and two mixing angles,  $\sin^2 \theta_{12}$  and  $\sin^2 \theta_{23}$ . For these observables we use the central values and  $2\sigma$  errors from Ref. [26]. The other observables: neutrino masses, 1-3 mixing angle and the phase of the lepton

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<sup>9</sup>In principle, these parameters can be complex. We will nevertheless assume that they are real; hence there are no additional CP violating phases in the neutrino sector.

<sup>10</sup>Note, the two Majorana phases are in principle observable, for example, in neutrinoless double-beta decay [21], however, the measurement would be very difficult (perhaps too difficult [22]). If observable, this would increase the number of predictions to 8.

<sup>11</sup>The reader will find more details on the soft SUSY breaking parameters in the next section.

<sup>12</sup>The only difference is that in the present analysis we include all three families of fermions.

mixing matrix are predictions of the model. We will discuss these later.

### *Best fit*

In Table 1 we present the best fit for quark and lepton masses and mixing for the  $\chi^2$  function constructed from the 20 observables in the quark and lepton sectors and 3 gauge couplings. The input parameters for this fit are summarized at the top of the table. One can see that the charged fermion sector is fit very well with the largest contributions to  $\chi^2$  coming from  $m_d/m_s$  and  $\sin(2\beta)$ .<sup>13</sup>

The neutrino observables, on the other hand, are fit very close to their central values, giving a negligible contribution to  $\chi^2$ . Note, our model does not rely on *single* right-handed neutrino dominance. In the case of SRHND [7, 8, 9, 10, 11, 12] one obtains maximal atmospheric mixing angle as described (see Eqn. 3), but these models (and others) differ significantly in the way they obtain the large solar mixing angle. Many models actually abandon the hierarchical texture of the neutrino Yukawa matrix or they assume a different form for the right-handed neutrino mass matrix. In [14] it was shown that it is *impossible* to get the desired “mild” hierarchy in  $m_{\nu_2}/m_{\nu_3}$  when one of the right-handed neutrinos dominates in strictly hierarchical models. To fix this, for example in [27], the authors make the (1 2) element comparable to the (2 2) element. But the Yukawa matrix is no longer hierarchical as in Eqn. 1. It appears that the only possible way to get the correct neutrino masses, and at the same time have bi-large mixing with Yukawa matrices of the form in Eqn. 1, is for the contributions of  $M_{R_1}$  and  $M_{R_2}$  to be comparable (but with opposite sign). Note that the (2 3) block of the resulting  $M_\nu$  does not change when varying the relative contribution of  $M_{R_1}$  and  $M_{R_2}$ . In [14], a good fit was found with  $M_{R_1}$  contributing only -1.5 times more than  $M_{R_2}$ . In our model, for the best fit, we find that  $M_{R_1}$  contributes only -1.2 times more than  $M_{R_2}$ .<sup>14</sup> In summary, comparable contributions of  $M_{R_1}$  and  $M_{R_2}$  are essential for getting bi-large mixing and an “observed” neutrino mass hierarchy (neutrino mass squared differences interpreted as normal mass hierarchy) in hierarchical models with Yukawa matrices of the form Eqn. 1. Finally the contribution of  $M_{R_3}$  is, in a model independent analysis, not very constrained, since we do not know the mass of the lightest neutrino. However, for the best fit in our model it turns out that all three right-handed neutrinos contribute comparably, and there is no single right-handed neutrino dominance. Indeed, forcing the contribution of  $M_{R_3}$  to be negligible (by pushing it all the way to the GUT scale) makes the fit worse, but does not change the fact that we would have bi-large mixing.

The best fit to fermion masses prefers the region of SUSY parameter space characterized by  $\mu$ ,  $M_{1/2} \ll m_{16}$  and  $-A_0 \simeq \sqrt{2}m_{10} \simeq 2m_{16}$ , where  $M_{1/2}$  is the universal gaugino mass,  $m_{10}$  is the universal Higgs mass,  $m_{16}$  is the universal squark and slepton

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<sup>13</sup>The problem associated with  $m_d/m_s$  is directly connected with the Georgi-Jarlskog [28] factor of three explicit in our model, while the problem with  $\sin(2\beta)$  is a result of using factorizable phases in our Yukawa matrices [29]. S.R. thanks G.C. Branco for pointing out the origin of the latter problem.

<sup>14</sup>We define  $r_i = (|Y_n(1,i)|^2 + |Y_n(2,i)|^2 + |Y_n(3,i)|^2)/M_{R_i}$  as the contribution of  $M_{R_i}$  to  $M_\nu$ . This definition is not unique, there is no reason to take  $|Y_n(1,i)|^2, \dots$  because  $Y_n(1,i)^2, \dots$  is what appears in  $M_\nu$ . But the latter is complex, so for a rough order of merit we take the absolute values instead.

mass and  $A_0$  is the universal trilinear coupling.<sup>15</sup> This is required in order to fit the top, bottom and tau masses when the third generation Yukawa couplings unify [23]. Note, third generation Yukawa unification receives only small corrections from 2-3 mixing in our model. For completeness, predictions for squark, slepton and Higgs masses for the best fit are presented in Table 2. However, it should be emphasized that the  $\chi^2$  function is not very sensitive to changes in the SUSY parameters, as long as the relations discussed above are approximately satisfied.<sup>16</sup> As a consequence of these relations we expect heavy first and second generation squarks and sleptons, while the third generation scalars are significantly lighter (with the stop generically the lightest). In addition, charginos and neutralinos are typically the lightest superpartners. We predict values of  $\tan\beta \sim 50$  and a light Higgs mass of order 120 GeV. The specific relations between the SUSY breaking parameters also leads to an interesting prediction for the process  $B_s \rightarrow \mu^+\mu^-$  with branching ratio in the region currently being explored at the Tevatron.<sup>17</sup> Furthermore, the neutralino relic density obtained for our best fit parameters is consistent with WMAP data [24] and direct neutralino detection is possible in near future experiments. Finally, this region maximally suppresses the dimension five contribution to proton decay [30] and suppresses SUSY flavor and CP violation in general. For a detailed analysis of SUSY and Higgs spectra and related phenomenology see Ref. [23] and [24].

#### *Additional predictions*

Finally let us discuss the predictions in the lepton sector, summarized in Table 3. At this point we would like to emphasize, that none of these observables were included in the  $\chi^2$  analysis and no parameters were constrained or tweaked in order to get the values we present. Therefore these observables are genuine predictions of the model. However, it should be noted that slightly different values of some of these observables might be obtained without significant changes in  $\chi^2$ . We did not study in detail to what extent each of the observables can be modified without significantly worsening the fit.

The only mixing angle in the lepton sector, which so far has not been measured, is 1-3 mixing. The  $2\sigma$  upper bound from the global fit to neutrino data (with Chooz and solar+KamLAND data contributing comparably to the fit) is [26]:

$$\sin^2 \theta_{13} < 0.031. \quad (25)$$

We obtain the best fit value given by

$$\sin^2 \theta_{13} \simeq 0.0024 \text{ (or } \sin^2 2\theta_{13} \simeq 0.01). \quad (26)$$

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<sup>15</sup>In addition, we require non-universal Higgs masses at the GUT scale. For the best fit we need  $1/2(m_{H_d}^2 - m_{H_u}^2)/m_{10}^2 \sim .07$ . Note this is significantly smaller than needed in the past [23]. That is because the RGE running of neutrino Yukawas from  $M_G$  to the heaviest right handed neutrino has been included self-consistently. As noted in [23], such running was a possible source for Higgs splitting. Evidently it can not be the only source.

<sup>16</sup>Note, the three input parameters ( $\mu$ ,  $M_{1/2}$ ,  $m_{16}$ ) are not varied when minimizing  $\chi^2$ . Moreover,  $\chi^2$  is basically flat with increasing  $m_{16}$ .

<sup>17</sup>This process is sensitive to the CP odd Higgs mass,  $m_A$ , which can be adjusted in theories with non-universal Higgs masses.

This is well below the upper bound and unfortunately too small to be seen in current or near future experiments. In particular, after three years of running the Double Chooz experiment will only probe down to  $\sin^2 2\theta_{13} \geq 0.03$  [31] and the maximal sensitivity goal for other future reactor experiments is just at the border of observability with  $\sin^2 2\theta_{13} \geq 0.01$  [32]. On the other hand, long baseline accelerator experiments, such as NUMI and T2K, may be able to probe the best fit value [33].

CP violation in the lepton sector given by, for example, the Jarlskog parameter,

$$J = \text{Im}(U_{e1}U_{e2}^*U_{\mu 1}^*U_{\mu 2}) = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin(\delta), \quad (27)$$

(where we use the standard parametrization, Eqn. 17, with the abbreviated notation  $s_{ij} \equiv \sin \theta_{ij}$  and  $c_{ij} \equiv \cos \theta_{ij}$ ), is proportional to  $\sin \theta_{13}$ . In spite of the fact that the best fit suggests a large CP violating phase  $\delta \sim \pi/2$ , the smallness of  $\sin \theta_{13}$  results in  $J \sim 0.01$ . CP violation of this magnitude may be observable at long baseline experiments. For example, the JPARC-SK experiment has a potential sensitivity to  $\sin^2 2\theta_{13} < 1.5 \times 10^{-3}$  and  $\delta \sim \pm 20^\circ$  and a comparable sensitivity is expected from the ‘‘Off-axis NUMI’’ proposal [34].

Majorana neutrino masses are, in principle, observable in processes like neutrinoless double-beta decay. The effective mass

$$\langle m_{\beta\beta} \rangle = \left| \sum_i U_{ei}^2 m_{\nu_i} \right| = \left| \sum_i |U_{ei}|^2 m_{\nu_i} e^{i\alpha'_i} \right| \quad (28)$$

(where  $\alpha'_i = \alpha_i + 2\delta$ ,  $i = 1, 2$  [35]) is predicted to be of order  $2 \times 10^{-4}$  eV which is too low to see in near-future experiments [36, 34]. This is a consequence of the Majorana phases  $\alpha_1$  and  $\alpha_2$  being almost opposite (see Table 3). The contribution of  $m_{\nu_1}$  and  $m_{\nu_2}$  is larger than  $m_{\nu_3}$  by an order of magnitude and, due to the opposite Majorana phases, these tend to cancel. This is also evident from the prediction for the effective electron-neutrino mass observable, relevant for the analysis of the low energy beta decay of tritium. This mass parameter is unaffected by Majorana phases and is predicted to be an order of magnitude larger. The observable,

$$m_{\nu_e}^{eff} = \left( \sum_i |U_{ei}|^2 m_{\nu_i}^2 \right)^{1/2} \quad (29)$$

is predicted to be of order  $6 \times 10^{-3}$  eV. The current experimental limit is  $m_{\nu_e}^{eff} \lesssim 2.5$  eV with the possibility of future experiments, such as KATRIN, reaching bounds on the order of 0.35 eV [34]. Unfortunately, both mass parameters may be unobservable by presently proposed experiments.

The best fit values of the heavy right-handed neutrino masses, defined in Eqn. 15, are given by

$$M_{R_1}, M_{R_2}, M_{R_3} \approx 10^{10}, 10^{12}, 7 \times 10^{14} \text{ GeV} \quad . \quad (30)$$

The lightest Majorana neutrino,  $R_1$ , is responsible for leptogenesis. The lepton number asymmetry parameter is given by [37, 38]

$$\epsilon_1 = -\frac{3}{16\pi} \frac{1}{(Y_\nu^\dagger Y_\nu^*)_{11}} \sum_{i=2,3} \text{Im}\{([Y_\nu^\dagger Y_\nu]_{1i})^2 \frac{M_{R_1}}{M_{R_i}}\} \quad (31)$$

where  $Y_\nu$  is the Dirac neutrino Yukawa matrix<sup>18</sup>. This formula is only valid in the limit  $M_{R_1} \ll M_{R_2}, M_{R_3}$ . An acceptable baryon asymmetry, obtained via leptogenesis, requires values of  $\epsilon_1 \sim \mathcal{O}(10^{-6})$ . In comparison, for the relevant parameter for non-thermal leptogenesis we find  $4 \times \epsilon_1 \approx -4.6 \times 10^{-7}$ . Note that the baryon asymmetry obtained from leptogenesis comes via electroweak baryon and lepton number violating interactions preserving B - L. Hence the resultant baryon asymmetry  $N_B \propto -N_L \propto -\epsilon_1$ . *Hence we even obtain the correct sign.*<sup>19</sup>

For the case of thermal leptogenesis, the relevant asymmetry parameter is  $8 \times \epsilon_1 \approx -9 \times 10^{-6}$ .<sup>20</sup> We note that the values for the parameters  $M_{R_1} \approx 10^{10}$  GeV and  $m_{\nu_1} \approx 4 \times 10^{-3}$  eV are in the regime where the cosmological baryon asymmetry only depends on neutrino properties [38].<sup>21</sup> It does not depend on the initial neutrino abundance nor the initial baryon asymmetry generated by other mechanisms. On the other hand, thermal leptogenesis requires the reheat temperature after inflation,  $T_R > M_{R_1} \approx 10^{10}$  GeV. Such a high reheat temperature will generate a gravitino problem, unless the gravitinos are heavy with mass greater than  $\mathcal{O}(10 \text{ TeV})$ .

## Conclusions

We construct a simple  $SO(10)$  SUSY GUT with  $D_3$  family symmetry and an unbroken low energy R parity. The model describes fermion mass matrices with 14 parameters and gives excellent fits to 20 observable masses and mixing angles in both quark and lepton sectors, giving 6 predictions. Bi-large neutrino mixing is obtained with hierarchical quark and lepton Yukawa matrices; thus avoiding the possibility of large lepton flavor violation. The model naturally predicts small 1-3 neutrino mixing,  $\sin \theta_{13} \simeq 0.05$ , and a CP violating phase  $\delta$  close to  $\pi/2$ . Among other interesting predictions is a tiny effective Majorana mass for neutrinoless double-beta decay. Leptogenesis is also possible with the decay of the lightest right-handed neutrino giving an acceptable CP violating asymmetry  $\epsilon_1$  of order  $10^{-6}$ , *and with the correct sign for the resultant baryon asymmetry*. We also note that similar models with the non-abelian symmetry groups  $SU(2)$  or  $D_4$ , instead of  $D_3$ , can be constructed [13].

The model presented in this letter provides an excellent benchmark for testing supersymmetric GUTs. At the very least, this model provides a phenomenological ansatz for fermion Yukawa matrices at the GUT scale, which fits low energy fermion masses and

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<sup>18</sup>This formula is valid for standard model neutrinos. In the case of non-thermal leptogenesis in supersymmetric theories, the above value of  $\epsilon_1$  is multiplied by an additional factor of 4, taking into account right-handed neutrino decays to leptons (or sleptons) in the final state with ordinary particles (and superpartners) in loops.

<sup>19</sup>In fact, we can obtain either sign. We obtain the opposite sign with a small increase of  $\chi^2$ .

<sup>20</sup>Since we now also include the asymmetry due to sneutrino decays.

<sup>21</sup>S.R. would like to thank W. Buchmüller for emphasizing this point.

Table 1: **The best fit for fermion masses and mixing angles.**

Initial parameters:  $(1/\alpha_G, M_G, \epsilon_3) = (24.98, 2.77 \times 10^{16} \text{ GeV}, -3.2 \text{ } \%),$   
 $(\lambda, \epsilon, \sigma, \tilde{\epsilon}, \rho, \epsilon', \xi) = (0.64, 0.046, 0.83, 0.011, -0.053, -0.0036, 0.12),$   
 $(\Phi_\sigma, \Phi_{\tilde{\epsilon}}, \Phi_\rho, \Phi_\xi) = (0.618, 0.411, 0.767, 3.673) \text{ rad},$   
 $(m_{16}, M_{1/2}, A_0, \mu(M_Z)) = (3500, 450, -6888.3, 247.9) \text{ GeV},$   
 $((m_{H_d}/m_{16})^2, (m_{H_u}/m_{16})^2, \tan \beta) = (2.00, 1.71, 49.98)$   
 $(M_{R_3}, M_{R_2}, M_{R_1}) = (5.8 \times 10^{13} \text{ GeV}, -9.3 \times 10^{11} \text{ GeV}, 1.1 \times 10^{10} \text{ GeV})$

| Observable<br>(masses in GeV) | Data ( $\sigma$ )  | Theory  | Pull        |
|-------------------------------|--------------------|---------|-------------|
| $G_\mu \times 10^5$           | 1.16637 (0.1%)     | 1.16635 | < 0.01      |
| $\alpha_{EM}^{-1}$            | 137.036 (0.1%)     | 137.031 | < 0.01      |
| $\alpha_s(M_Z)$               | 0.1187 (0.002)     | 0.1184  | 0.02        |
| $M_t$                         | 174.3 (5.1)        | 175.36  | 0.04        |
| $m_b(M_b)$                    | 4.25 (0.25)        | 4.252   | < 0.01      |
| $M_b - M_c$                   | 3.4 (0.2)          | 3.513   | 0.32        |
| $m_c(m_c)$                    | 1.2 (0.2)          | 1.03    | 0.72        |
| $m_s$                         | 0.105 (0.025)      | 0.114   | 0.13        |
| $m_d/m_s$                     | 0.0521 (0.0067)    | 0.0627  | <b>2.53</b> |
| $Q^{-2} \times 10^3$          | 1.934 (0.334)      | 1.763   | 0.26        |
| $M_\tau$                      | 1.777 (0.1%)       | 1.777   | < 0.01      |
| $M_\mu$                       | 0.10566 (0.1%)     | 0.10566 | < 0.01      |
| $M_e \times 10^3$             | 0.511 (0.1%)       | 0.511   | < 0.01      |
| $V_{us}$                      | 0.22 (0.0026)      | 0.2192  | 0.09        |
| $V_{cb}$                      | 0.0413 (0.0015)    | 0.0407  | 0.15        |
| $V_{ub}$                      | 0.00367 (0.00047)  | 0.00332 | 0.56        |
| $V_{td}$                      | 0.0082 (0.00082)   | 0.00819 | < 0.01      |
| $\epsilon_K$                  | 0.00228 (0.000228) | 0.00238 | 0.19        |
| $\sin(2\beta)$                | 0.736 (0.049)      | 0.6757  | <b>1.51</b> |
| $\Delta m_{31}^2 \times 10^3$ | 2.3 (0.6)          | 2.407   | 0.03        |
| $\Delta m_{21}^2 \times 10^5$ | 7.9 (0.6)          | 7.866   | < 0.01      |
| $\sin^2 \theta_{12}$          | 0.295 (0.045)      | 0.2851  | 0.05        |
| $\sin^2 \theta_{23}$          | 0.51 (0.13)        | 0.546   | 0.08        |
| TOTAL $\chi^2$                |                    |         | <b>6.70</b> |

Table 2: Predictions for SUSY and Higgs spectra for the best fit given in Table 1.

| Particle         | Mass (GeV) |
|------------------|------------|
| $h$              | 121.6      |
| $H$              | 495.5      |
| $A^0$            | 499.9      |
| $H^+$            | 411.5      |
| $\chi_1^0$       | 174.1      |
| $\chi_2^0$       | 241.7      |
| $\chi_1^+$       | 230.7      |
| $\tilde{g}$      | 1181.0     |
| $\tilde{t}_1$    | 299.4      |
| $\tilde{b}_1$    | 748.4      |
| $\tilde{\tau}_1$ | 1128.1     |

Table 3: Predictions for neutrino masses,  $\sin \theta_{13}$  and CP violation in the lepton sector for the best fit given in Table 1.

|                                       |                        |
|---------------------------------------|------------------------|
| $m_{\nu_3}$ (eV)                      | 0.0492                 |
| $m_{\nu_2}$ (eV)                      | 0.0096                 |
| $m_{\nu_1}$ (eV)                      | 0.0037                 |
| $\sin^2 \theta_{13}$                  | 0.0024                 |
| $J$                                   | 0.0107                 |
| $\sin \delta$                         | 0.98                   |
| $\alpha_1$ (rad)                      | -1.286                 |
| $\alpha_2$ (rad)                      | 1.821                  |
| $\langle m_{\beta\beta} \rangle$ (eV) | 0.00021                |
| $m_{\nu_e}^{eff}$ (eV)                | 0.0065                 |
| $\epsilon_1$                          | $-1.16 \times 10^{-7}$ |

mixing angles. There are several possible directions for future research which should be explored. First, the model will make predictions for flavor violating processes, such as  $\mu \rightarrow e \gamma$ . This process, as well as other lepton flavor violating processes, will be tested to much higher accuracy in the future [40]. Secondly, as discussed in the text we obtain acceptable values for the lepton number asymmetry parameter relevant for leptogenesis,  $\epsilon_1$ . Thus it would be interesting to see if non-thermal leptogenesis via inflaton decay, as discussed by several authors (see [41] and references therein), can be incorporated into this theory.

Finally, can this theory be derived from a more fundamental starting point such as string theory. Although we have not shown that this is possible, it is likely that it may be re-written as a five dimensional orbifold GUT (see [42]).

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